

Lecture 1

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The Derivative

1. Choose an interval

2. Find the raw change

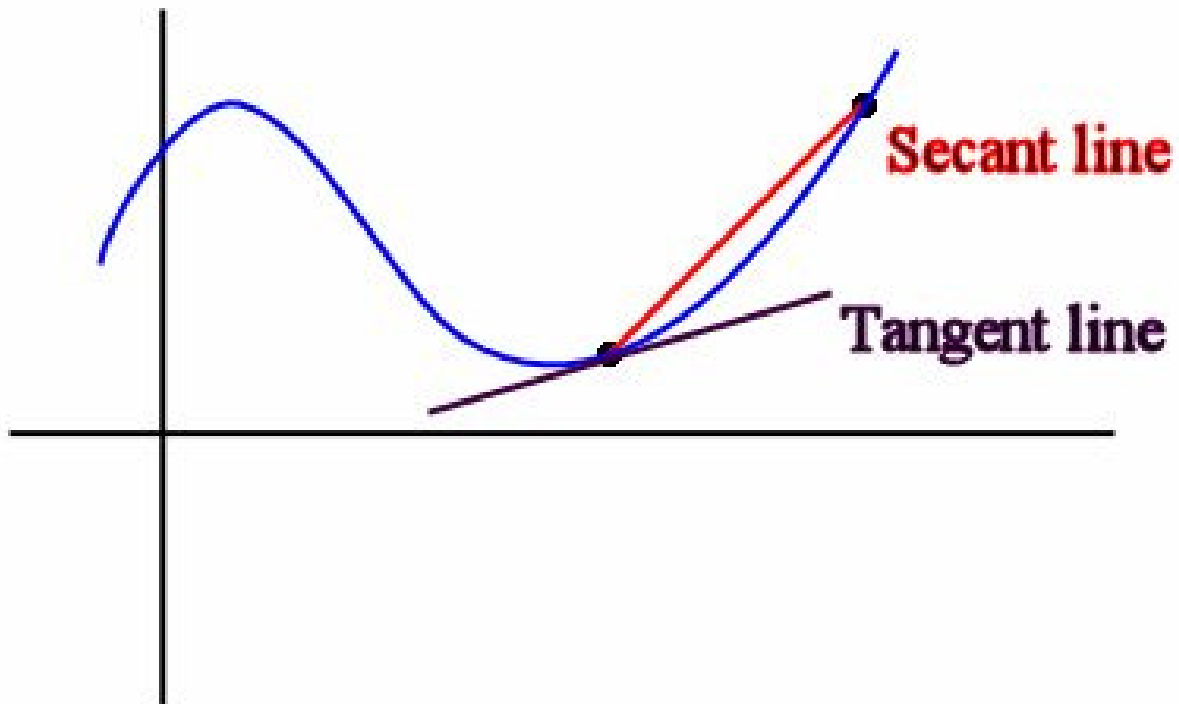
$$f'(x) = \lim_{dx \rightarrow 0} \frac{f(x + dx) - f(x)}{dx}$$

4. Make your model perfect

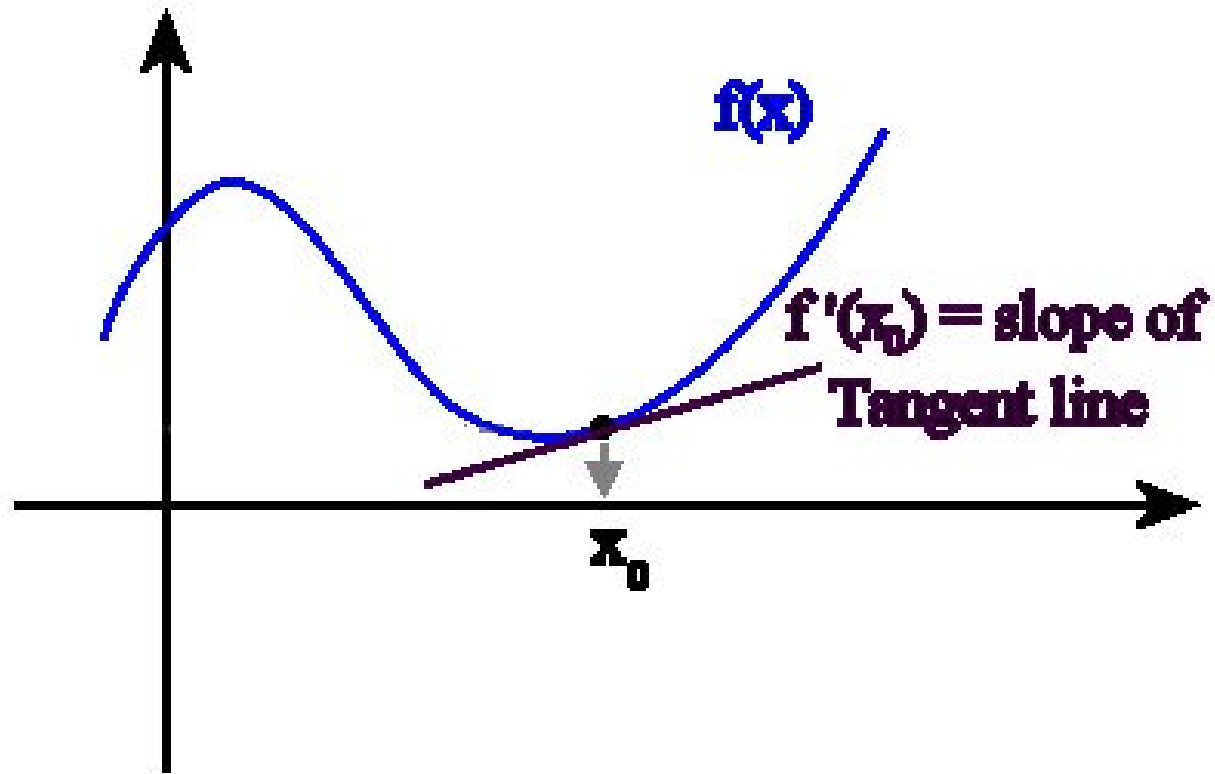
3. Find the rate of change

Limit

$$dx \rightarrow 0$$



Slope of Tangent



Convention

$$y = f(x)$$

$$\frac{dy}{dx} = \frac{df(x)}{dx} = f'(x)$$

$\frac{dy}{dx}$ has a special meaning

*you cannot cancel the d from
numerator and denominator*

Memorize This

$$\frac{d}{dx}(\mathbf{constant}) = 0$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}x^k = kx^{k-1}$$

$$\frac{d}{dx}a^x = (\ln a)a^x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\log_a x = \frac{1}{x(\ln a)}$$

Sum and Difference Rule

α β γ *are constants*

$$\frac{d}{dx}(\alpha f + \beta g - \gamma h) = \alpha \frac{df}{dx} + \beta \frac{dg}{dx} - \gamma \frac{dh}{dx}$$

Inverse Functions

$$y = \sin^{-1} x \quad \text{find } \frac{dy}{dx}$$

$$x = \sin y$$

$$\frac{dx}{dy} = \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

$$\text{Now find } \frac{d}{dx} \cos^{-1} x$$