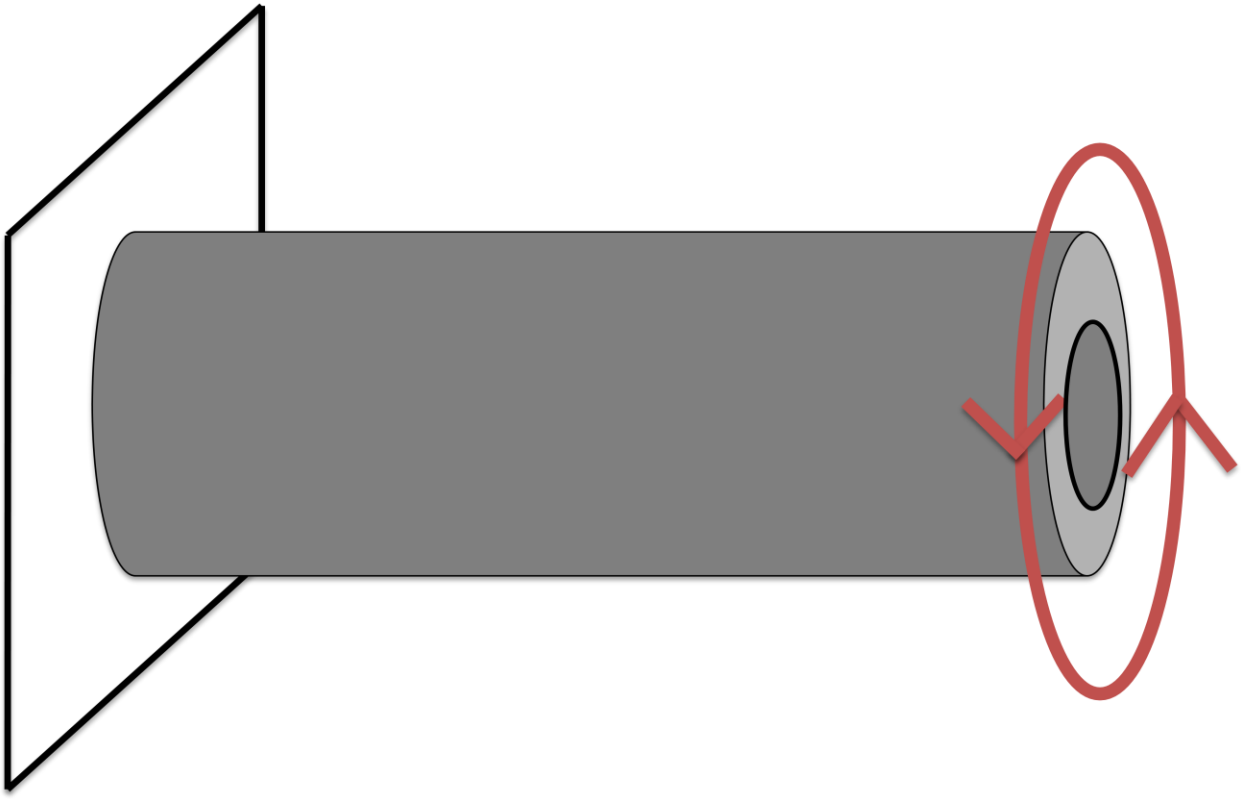
Ambar Mitra

Example Problem:

A composite cantilever bar (L=100in) is loaded with a counter-clockwise torque of 2000 in-lbs. on its free, right end. The outer section of the bar is steel (G=11.5e6 psi) and has an outside diameter of 2 in. The inner section of the beam is aluminum (G=3.70e6 psi) and has a diameter of 1.5 in.



Solution:

$$J_{in} = \frac{\pi {d_{in}}^4}{32} = \frac{\pi * (1.5in)^4}{32} = 0.497in^4$$

$$J_{out} = \frac{\pi({d_{out}}^4 - {d_{in}}^4)}{32} = \frac{\pi * ((2in)^4 - (1.5in)^4)}{32} = 1.074in^4$$

$$T = T_{in} + T_{out}$$

$$\theta_{inner} = \theta_{outer}$$

$$\frac{T_{in} L}{J_{in} G_{in}} = \frac{T_{out} L}{J_{out} G_{out}}$$

$$T_{in} = T_{out} \frac{J_{in} G_{in}}{J_{out} G_{out}}$$

$$T = T_{out} + T_{out} \frac{J_{in} G_{in}}{J_{out} G_{out}} = T_{out} \left(1 + \frac{J_{in} G_{in}}{J_{out} G_{out}}\right)$$

$$T_{out} = \frac{T}{1 + \frac{J_{in} G_{in}}{J_{out} G_{out}}}$$

$$T_{out} = \frac{2000\ in \cdot lb}{1 + \frac{0.497\ in^4\ 3.70e6\ psi}{1.074\ in^4 11.5e6\ psi}} = 1740.82\ in \cdot lb$$

$$T_{in} = T - T_{out} = 259.184\ in \cdot lb$$

$$\tau_{out} = \frac{.5 T_{out}\, d_{out}}{J_{out}} = \frac{.5 * 1740.82\ in \cdot lb * 2.0in}{1.074 in^4} = 1620.87\ psi$$

$$\tau_{in} = \frac{.5 T_{in}\, d_{in}}{J_{in}} = \frac{.5 * 259.184\ in \cdot lb * 1.5in}{0.497 in^4} = 391.123\ psi$$

$$\theta = \frac{T_{out}\ L}{J_{out}\ G_{out}} = \frac{1740.82\ in \cdot lb * 100\ in}{1.074 in^4 11.5e6\ psi} = 0.0141 \text{ rad}$$